PROBABILITY:

The statistical technique of studying the chances of occurrence or non-occurrence/happening or not happening of certain phenomenon or events is called probability.

Terminology used in probability:

Random experiment or trail: An experiment whose outcome cannot be predicted is called random experiment or trail. Tossing a coin, rolling a die etc are examples.

Outcome or output: The results of the random experiments are called outcome or output.

Sample space: The collection of all possible outcome of the random experiment is called sample space and it is denoted by S. When a die is rolled then S = (1, 2, 3, 4, 5, 6), tossing a coin then S = (H, T) are examples of sample space.

Exhaustive cases: Total number of possible outcomes in any trail is called exhaustive cases or events. For example in tossing a coin there are 2 exhaustive cases, in rolling a fair die there are six exhaustive cases.

Impossible event: The empty set ϕ, is a sub set of sample space so it is called impossible event.

Equally likely events: Outcomes of a trail are said to be equally likely if none of them is expected to occur in preference to other. In tossing a fair coin all the outcomes are equally likely.

Mutually exclusive events: Two or more events are said to be mutually exclusive if they cannot happen at the same time. For example in tossing a coin the event head and tail are mutually exclusive.

Independent event: Two or more events are said to be independent if the probability of occurrence of one does not affect the probability of occurrence of other. If we draw a card from a pack of well shuffled cards and replace it before drawing the second card, then the result of second draw is independent of the first draw. But however, if the first card drawn is not replaced then the second draw is dependent to the first draw.

Favorable events: The number of cases favorable to an event in a trail is the number of outcomes which entail the happening of the event. For example, in tossing two coins, the number of cases favorable to the event exactly one head is 2 i.e. HT, TH and for getting at least one head is 3 i.e. HH, HT, TH.

Complementary event: Events A and A^c or $\bar{A}$ which are defined in sample space are said to be complementary events if A^c has not any sample point of A. If S = {1, 2, 3, 4, 5, 6}, A = {1, 3, 5} then complement of A is S – A = A^c = {2, 4, 6}.

Counting Process:

It is quite difficult to determine the set of all possible outcomes of a random experiment by direct enumeration. There are three different ways to determine the total possible ways of a random experiment.

I) Multiplicative rule: Let a random experiment is performed in sequence of n ways in which first trail has k_1 possibilities, second trail has k_2 possibilities and so on.

Then total possible outcomes of all random experiment = $k_1.k_2.k_3$...........

Ex: Let a coin is tossed and a die is rolled simultaneously, then how many possibilities are there?

Here, when tossing a coin (k_1) = 2

When rolling a die (k_2) = 6

Then total possible outcomes = $k_1 . k_2$ = 2.6 = 12

Above example can also be present in tree diagram.

II) Combination: The combination of n things taken r at a time is given by

$$n_{C_r} = \frac{n!}{(n-r)!r!},$$

Ex: The government has decided to lunch new hospitals in 5 cities from total of 20 cities. How many ways government can select the cities?

Solution: Here n = 20, r = 5, then using the relation of combination we get

$$20_{C_5} = {20!}\big/{(20-5)!5!} = 15504$$

III) Permutation: The permutation of n things taken r at a time denoted by n_{P_r} is an order arrangement and is given by

$$n_{P_r} = \frac{n!}{(n-r)!}$$

Ex: Find the number of permutation of 4 different objects taken 2 at time.

Solution: here n = 4, r = 2

Then $4_{P_2} = {4!}\big/{(4-2)!} = 12$

Hence each of the arrangements which can be made by taking some or all of a number of things is called permutation and it consider order of the arrangement. Each of the groups or selections which can be made by taking some or all of things is called combinations i.e. it does not consider order of the objects. Thus the permutation which can be made by taking the letters a, b ,c, d two at a time are: ab, ac, ad, bc, bd, cd, ba, ca, da, cb, db, dc each of these presenting arrangement of two letters.

The combination which can be made by taking the letters a,b,c, d two at a time are ab, ac, ad, bc, bd, cd.

Various approaches of probability

There are three different approaches of probability, they are:

1) Mathematical/Priori or classical approach:

If there are n exhaustive, mutually exclusive and equally likely outcomes of a random experiment and m of them are favorable to an event A. Then probability of happening of event A is denoted by P(A) and is defined as

P(A) = favorable number of cases/total number of cases = m/n

Remarks:

i) Since m and n are non-negative integer, so P(A) $\geq$ 0 and m $\leq$ n, so P(A) $\leq$ 1. Hence probability of any event is always lying between 0 $\leq$ P(A) $\leq$ 1.

ii) P(A) is known as probability of success and is usually written as p and P($\bar{A}$) is known as probability of failure and is denoted by q. Therefore P(A) + P($\bar{A}$) =1 or p + q =1.

2) Statistical or empirical approach:

Classical approach fails to determine probability when outcome of the trails are not equally likely and when the number of trails are performed in n repetitions of random experiment to solve it statistical or empirical approach of probability is developed.

Suppose that an event A occurs ‘m’ times in ‘n’ repetitions of a random experiment, in limiting case when n becomes sufficiently large, then probability of happening event A is defined as

P(A) = $\lim_{n\to\infty} \frac{m}{n}$

3) Modern or axiomatic approach to probability:

Let S be the sample space of random experiment and A be any event in S, then P is probability function which is defined in S and the function P(A) satisfying the following axioms.

i) P(A) is defined, real and non negative i.e. P(A) ≥ 0

ii) The total probability P(S) = 1

iii) If $A_1, A_2, \ldots\ldots, A_n$ is any finite or infinite sequence of disjoint events of S then

$$P(A_1 \cup A_2 \cup, \ldots\ldots, A_n) = P(A_1) + P(A_2) + \ldots\ldots + P(A_n)$$

The above axioms are known as axioms of positiveness, certainty and additive respectively.

Examples:

1)A bag contains 7 white and 3 black balls what is the probability of getting a white ball?

Solution: Total number of cases (n) = 7 + 3 = 10

Favourable number of case of getting a white ball = 7

$\therefore$ Probability of drawing a white ball = 7/10 = 0.7

2) From a group of 3 doctors, 4 nurses and 5 helpers, a committee of 4 persons is selected at random. Find the probability that the committee will consists of a) 2 doctors and 2 nurses b) 1 doctors, 1 nurse and 2 helpers c) 4 helpers

Solution: Total possible cases (n) = 3 + 4 + 5 =12 persons is 12_{C_4} =12!/(12-4)! 4! = 495

a) From 3 doctors and 4 nurses, 2 doctors and 2 nurses can be selected as (m) = $3_{C_2}.4_{C_2}$ = 3.6 = 18 ways

$\therefore$ P(2 doctors and 2 nurses) = m/n = 18/495 = 0.036

b)From 3 doctors, 4 nurses and 5 helpers, 1 doctor 1nurse and 2 helper can be selected as (m) = $3_{C_1}.4_{C_1}.5_{C_2}$ = 3.4.10 =120 ways

$\therefore$ P (1 doctor, 1 nurse and 2 helper) = m/n = 120/495 = 0.242

c) From 5 helper, 4 helper can be selected as (m) = 5_{C_4} = 5 ways

$\therefore$ P (5 helpers) = m/n = 5/495 = 0.0101

Laws of probability

1) The probability of impossible event is zero i.e, P(ϕ) = 0

2) The probability of complementary event $\bar{A}$ of A is given by: P($\bar{A}$) = 1 – P (A)

3) Additive rule of probability:

a) When the events are mutually exclusive: Let A and B be two mutually exclusive events with respective probabilities P(A) and P(B) then probability of occurrence of at least one event i.e. event A or B is given by: $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$

b) When the events are not mutually exclusive: Let A and B are not two mutually exclusive events with respective probabilities P(A) and P(B) then probability of occurrence of at least one event i.e. event A or B is given by:

$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) – P(A \cap B)$

Ex: 1) A box contains 30 blood samples numbered 1 to 30. One blood sample is drawn at random; find the probability that number of blood sample drawn will be multiple of 5 or 7?

Solution: let A be the event of getting the number of multiples of 5 (i.e. 5, 10, 15, 20, 25, 30)

Number of multiple of 5 (m) = 6,   total no. of blood sample (n) = 30

$\therefore P(A) = m/n = 6/30 = 0.2$

Similarly B be event of getting multiple of 7 (i.e. 7, 14, 21, 28) . Therefore m = 4

$P(B) = m/n = 4/30 = 0.13$

Here A and B are two mutually exclusive events therefore the required probability is given by

$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) = 0.2 + 0.13 = 0.33$

Ex.2) A box contains 30 blood samples numbered 1 to 30. One blood sample is drawn at random. Find the probability that the number of blood sample will be multiple of 5 or 10.

Solution: Let A be the event of getting the number of multiple of 5

i.e. $P(A) = m/n = 6/30 = 0.2$

B be the event of getting the number of multiple of 10 (i.e. 10, 20, 30). Therefore $m = 3$

$P(B) = m/n = 3/30 = 0.1$

Here 10, 20 and 30 are multiples of both 5 and 10. So probability of occurrence of both A and B = $P(A \cap B) = 3/30 = 0.1$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.2 + 0.1 - 0.1 = 0.2$

4) Multiplicative rule of probability:

a) When the events are independent: Let A and B be two independent events with their respective probabilities P(A) and P(B) then their simultaneous occurrences is given by

$P(A \text{ and } B) = P(A \cap B) = P(A).P(B)$

b) When the events are dependent: Let A and B are two dependent events, then probability of occurrence of one event A is affected by the probability of occurrence of B and vice versa. This relationship is called conditional probability.

The conditional probability of A that B has already happened is denoted by P(A/B) i.e. probability A given B is defined as

P(A/B) = $\frac{P(A \cap B)}{P(B)}$, P(B) > 0

Similarly P(B/A) = $\frac{P(A \cap B)}{P(A)}$, P(A) > 0

Ex: A card is drawn from a deck and replaced, and then a second card is drawn. Find the probability of getting i) a king and then a king ii) a king and than a queen iii) a red card and than a black card

Solution: Since after first draw the card is replaced, and made second draw so it is the case of independent events.

i) The probability of drawing king and then king

$P(k_1 \cap k_2) = P(k_1).P(k_2)$ = 4/52 . 4/52 = 1/169

ii) P(K$\cap$Q) = P(K).P(Q) = 4/52 . 4/52 = 1/169

iii) P(R$\cap$B) = P(R).P(B) =26/52 . 26/52 = ¼

Ex: A box contains 8 microscopes of which 3 are defective. Two microscopes are drawn at random one after the other without replacement. Find the probability that both the microscopes drawn are i) defective ii) non defective.

i) Let A be the event of drawing defective microscope in first draw and B be the event of drawing defective microscope in the second draw.

P(A) = 3/8 , P(B/A) = (3-1)/(8-1) = 2/7

Probability that both microscopes drawn are defective

$P(A \cap B) = P(A).P(B/A)$ = 3/8.2/7 = 0.11

ii) Let A be the event of drawing non-defective microscope in first draw and B be the event of drawing non-defective microscope in the second draw.

P(A) = 5/8, P(B/A) = (5-1)/(8-1) = 4/7

Probability that both microscopes drawn are non defective

$P(A \cap B) = P(A).P(B/A)$ = 5/8 . 4/7 = 0.36

Ex: The probability that a male patients selected at random from the current residence of a certain hospital is 0.6. The probability that the patient will be a male who is in for surgery is 0.2. A patient randomly selected from a current residence is found to be a male, what is the probability that the patients is in the hospital for surgery?

Solution: Probability of selecting a male patients P(M) = 0.6

The probability of patient who is male and is in for surgery $P(M \cap S)$ = 0.2

P(S/M) = ?

P(S/M) = $P(M \cap S)$/P(M) = 0.2/0.6 = 0.34

Ex: A speaks truth in 75% of the cases and B in 80% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact?

Solution: Let E_1 be the event that A speaks the truth and E_2 be the event that B speaks truth.

P(E_1) = 75% = 3/4, P($\bar{E}_1$) = 1 – 3/4 = 1/4

P(E_2) = 80% = 4/5,   P($\bar{E}_2$) = 1- 4/5 = 1/5

Let E be the event that A and B contradict each other, i.e. if one of them speaks the truth the other does not speaks truth. Thus

E = ($E_1 \cap \bar{E}_2$) or ($\bar{E}_1 \cap E_2$)

P(E) = P($E_1 \cap \bar{E}_2$) $\cup$ P($\bar{E}_1 \cap E_2$)

= P($E_1 \cap \bar{E}_2$) + P($\bar{E}_1 \cap E_2$) $\because$ ($E_1 \cap \bar{E}_2$) and ($\bar{E}_1 \cap E_2$) mutually exclusive

$\therefore$ P(E) = P(E_1) .P($\bar{E}_2$) + P($\bar{E}_1$) . P(E_2) = 3/4 . 1/5 + 1/4 . 4/5 = 0.35

Independence:

An event A is said to be independent of another event B if the occurrence or not occurrence of another event B does not affect the probability of the happening of the event A. Symbolically

P(A$\cap$B) =P(A).P(B)

If the conditional probability of A given B is equal to the unconditional probability of A, then A is said to be independent of B. Symbolically P(A/B) = P(A)

Ex: The probability that a man will be alive 45 years is 0.3 and probability that his wife will be alive 45 years is 0.4.Find the probability that for 45 years i) both will be alive ii) only man will be alive iii) only the women will alive iv) at least one of them will alive

Solution: Let A and B be two events which denote a man and his wife will alive for 45 years. Then P(A) = 0.3 and P(B) = 0.4

i) Both will alive 45 years P(A$\cap$B) =P(A).P(B) = 0.3 x 0.4 = 0.12

ii) Only man will alive 45 years P(A$\cap \bar{B}$) = P(A).P($\bar{B}$) = 0.3(1 – 0.4) = 0.18

iii) Only the woman will alive 45 years P($\bar{A}\cap$B) = P($\bar{A}$).P(B) = (1 – 0.3)0.4 = 0.28

iv)Probability at least one of them will alive 45 years is

P(A$\cup$ B) = P(A) + P(B) - P(A$\cap$B)

= P(A) +P(B) – P(A).P(B) = 0.3 + 0.4 – 0.3x0.4 = 0.58

Alternately,

P(at least one will live 45 years) = 1 –{(1-P(A))(1-P(B)} = 1 – {P($\bar{A}$).P($\bar{B}$)} = 1 – 0.7x0.6 = 0.58

Marginal probability and Joint probability

Joint probability refers to the phenomenon containing two or more than two events. Marginal probability refers to the probability of the simple events; it is also called as simple probability. Let X and Y be two discrete random variables, then the joint probability distribution function P(x_i, y_j) i = 1,2,.......,m j = 1,2,.......,n is called joint probability mass function if it satisfies i) P($x_i, y_j) \geq 0$ ii) $\sum_i \sum_j P(x_{i,} y_{j)}$ =1.

Let P(x_i, y_j) be the joint probability mass function of X and Y. Then marginal probability function of X is defined as

$$P_i = P(X = x_i) = \sum_j P(x_i, y_j) = P(x_i, y_1) + P(x_i, y_2) + ... + P(x_i, y_n)$$

Similarly, the marginal probability function of Y is defined as

$$P_j = P(Y = y_j) = \sum_{i=1}^{m} (x_i, y_j) = P(x_{1,} y_j) + P(x_2 y_j) + ... + P(x_m y_j)$$

Ex: The following contingency table represents the sex-wise knowledge of dental carries among the 300 school children studying in a certain school of Nepal. Using the information compute the marginal and joint probability.

	Knowledge on dental caries		Total
	yes	No	
Male	128	11	139
Female	153	8	161
Total	281	19	300

Solution: Computing joint probability

P(M and Y) = P(M$\cap$Y) =128/300 = 0.43,   P(F$\cap$Y) = 153/300 = 0.51

P(M$\cap$N) = 11/300 = 0.04,   P(F$\cap$N) = 8/300 = 0.02

Then marginal probability

P(M) = 139/300 = 0.47,   P(F) = 161/300 = 0.52

P(Y) = 281/300 = 0.94, P(N) = 19/300 = 0.06

Bayes's Theorem: Let $E_1, E_2, ..., E_n$ be mutually exclusive events of sample space S with $P(E_i) \neq 0,$ i = 1,2,.....,n then for any arbitrary event A which is subset of S such that P(A) > 0, we have

$$P(E_r / A) = \frac{P(E_r).P(A/E_r)}{\sum_{i=1}^{n} P(E_i).P(A/E_i)}, 1 \leq r \leq n$$

Bayes's theorem is one of the important applications of conditional probability which gives the information about the occurrence of one event to predict the probability of another event. This concept can be extended to revise probabilities based on new information and to determine the probability of particular effect was due to specific reason.

Prevalence and Incidence: Prevalence of a disease is the probability of having disease. It is the number of people with the disease divided by the number of people in the population. Incidence of a disease is the probability that a person without the disease will develop the disease during some specified interval of time. Prevalence provides an idea of current magnitude of the disease problem, where as incidence informs as to whether the disease problem is getting worse or not. Among them probability is also used to find the sensitivity, specificity, predictive value positive and negative as in the following example.

Ex: The table below summarizes the results of BCPF test of 1600 individuals.

Test result	Disease		
	Present(D)	Absent($\bar{D}$)	Total
positive (T)	76	224	300
negative ($\bar{T}$)	4	1296	1300
Total	80	1520	1600

Find sensitivity, specificity, predictive value positive and predictive value negative

Sensitivity: probability of positive test results given that patients have disease is given by

P(T/D) = P(T$\cap$D)/P(D) = $\frac{76/1600}{80/1600}$ = 0.94

Specificity: Probability of negative test results given that patients have no disease is

P($\bar{T}/\bar{D}$) = P($\bar{T} \cap \bar{D}$)/P($\bar{D}$) = $\frac{1296/1600}{1520/1600}$ = 0.84

Predictive value positive: Probability that patients have disease given a positive test result is

P(D/T) = $\frac{P(D).P(T/D)}{P(D).P(T/D)+P(\bar{D}).P(T/\bar{D})} = \frac{0.05x0.94}{0.05x0.94+0.14x0.94} = 0.25$

Predictive value negative: Probability that the patients have no disease given a negative test result

P($\bar{T}/\bar{D}$) = $\frac{P(\bar{D}).P(\bar{T}/\bar{D})}{P(\bar{D}).P(\bar{T}/\bar{D})+P(D).P(\bar{T}/D)}$ =

More examples on Bayes theorem:

A patients goes to see a doctor, the doctor perform a test with 99% reliability that, 99% of the people who are sick test positive and 99% of the healthy people test negative. The doctor knows that 1% of the people in the country are sick. Now the question is that if the patient test positive what are the probability the patient is really sick? The intuitive answer is 99%, but correct answer is 50%.

The solution to this question can easily be calculated using Bayes's theorem. Bayes, who lived from 1702 to 1761 stated that the probability you test positive AND are sick is the product of the likelihood that you test positive GIVEN that you are sick and the "prior" probability that you are sick (the prevalence in the population). Bayes's theorem allows one to compute a conditional probability based on the available information.

Bayes's Theorem
$P(A \mid B) = \dfrac{P(B \mid A)P(A)}{P(B)}$

Above explanation can be summarized with the help of the following table which illustrates the scenario in a hypothetical population of 10,000 people:

	Diseased	**Not Diseased**	
Test +	99	99	198
Test -	1	9,801	9,802
	100	9,900	10,000

What we want to know is P (A | B), i.e., the probability of disease (A), given that the patient has a positive test (B). We know that prevalence of disease (the unconditional probability of disease) is 1% or 0.01; this is represented by P(A). Therefore, in a population of 10,000 there will be 100 diseased people and 9,900 non-diseased people. We also know the sensitivity of the test is 99%, i.e., P(B | A) = 0.99; therefore, among the 100 diseased people, 99 will test positive. We also know that the specificity is also 99%, or that there is a 1% error rate in non-diseased people. Therefore, among the 9,900 non-diseased people, 99 will have a positive test. And from these numbers, it follows that the unconditional probability of a positive test is 198/10,000 = 0.0198; this is P(B).

Thus, P(A | B) = (0.99 x 0.01) / 0.0198 = 0.50 = 50%.

From the table above, we can also see that given a positive test (subjects in the Test + row), the probability of disease is 99/198 = 0.05 = 50%.

Another Example:

Suppose a patient exhibits symptoms that make her physician concerned that she may have a particular disease. The disease is relatively rare in this population, with a prevalence of 0.2% (meaning it affects 2 out of every 1,000 persons). The physician recommends a screening test that costs $250 and requires a blood sample. Before agreeing to the screening test, the patient wants to know what will be learned from the test, specifically she wants to know the probability of disease, given a positive test result, i.e., P(Disease | Screen Positive).

The physician reports that the screening test is widely used and has a reported sensitivity of 85%. In addition, the test comes back positive 8% of the time and negative 92% of the time.

The information that is available is as follows:

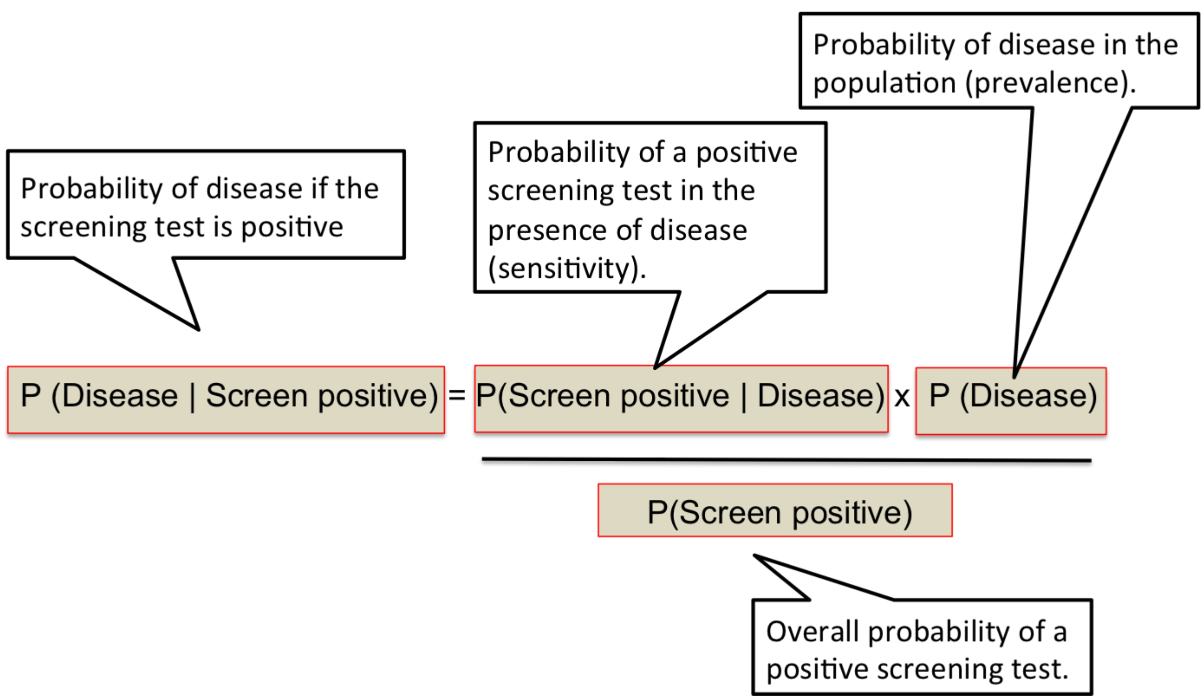
- P(Disease)=0.002, i.e., prevalence = 0.002
- P(Screen Positive | Disease)=0.85, i.e., the probability of screening positive, given the presence of disease is 85% (the sensitivity of the test), and
- P(Screen Positive)=0.08, i.e., the probability of screening positive overall is 8% or 0.08. We can now substitute the values into the above equation to compute the desired probability,

Based on the available information, we could piece this together using a hypothetical population of 100,000 people. Given the available information this test would produce the results summarized in the table below. Point your mouse at the numbers in the table in order to get an explanation of how they were calculated.

	Diseased	Not Diseased	
Test +	**170**	**7,830**	**8,000**
Test -	**30**	**91,970**	**92,000**
	200	**99,800**	100,000

The answer to the patient's question also could be computed from Bayes's Theorem

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$



We know that P(Disease)=0.002, P(Screen Positive | Disease)=0.85 and P(Screen Positive)=0.08. We can now substitute the values into the above equation to compute the desired probability, P(Disease | Screen Positive) = (0.85)(0.002)/(0.08) = 0.021.

If the patient undergoes the test and it comes back positive, there is a 2.1% chance that he has the disease. Also, note, however, that without the test, there is a 0.2% chance that he has the disease (the prevalence in the population). In view of this, do you think the patient have the screening test?

Another important question that the patient might ask is, what is the chance of a false positive result? Specifically, what is P(Screen Positive| No Disease)? We can compute this conditional probability with the available information using Bayes Theorem.

$$P(\text{Screen positive} \mid \text{No disease}) = \frac{(P(\text{No disease} \mid \text{Screen Positive}) \times P(\text{Screen positive})}{P(\text{No disease})}$$

P(Screen Positive | No Disease) = (1-0.021)(0.08)/(1-0.002) = 0.078.

Thus, using Bayes Theorem, there is a 7.8% probability that the screening test will be positive in patients free of disease, which is the false positive fraction of the test.

Exercise:

1. There are 3 public health officer, 4 doctors, 2 nurses and 1 lab technician. A committee of 4 among them is to be formed. Find the probability that the committee i) consists of one of each kind ii) consists of 2 doctors, 1 nurse and 1 lab technician.

2. The probability that male doctor selecting in a job as medical officer is 0.57 and that of female doctor is 0.43. What is the probability that at least one of them will be selected?

3. A subcommittee of 6 members is formed out of 7 doctors and 4 public health officers. Calculate the probability that the subcommittee will consists of i) exactly one public health specialist ii) at most one public health specialist iii) at least 5 doctors.

3. In a class of 32 students, it was found that 20 students secure grade A, 8 students secure grade B, and 4 students secure grade C in the previous exam. What is the probability of selecting a student who has i) either grade A or B ii) either grade B or C?

4. In a group of 15 persons, 8 suffers from fever, 10 suffers from cough and 6 suffers from both. A person is selected at random, what is the probability that he suffers from fever or cough.

5. Suppose that 3% of the people in the population of the adults having attempted suicide. It is also known that 20% of the populations are living below poverty level. If these two events are independent what is the probability that a person selected at random from the population will have attempted suicide and be living below the poverty level?

6. The probability that a man will be alive 25 year hence is 0.3 and the probability that his wife will be alive 25 years hence is 0.4. Find the probability that 25 years hence i) both will be alive ii) only the man will be alive iii) at least one of them alive. (Ans: 0.12, 0.18 and 0.58)

7. A problem of biostatistics is given to three students. A, B and C whose chances of solving it are 1/2, 1/3, and 1/4 respectively. Find the probability that i) all of them can solve the problem ii) none of them can solve the problem iii) problem will be solved. (1/24, 1/4 and 3/4)

8. From group of 8 children, 5 boys and 3 girls; 3 children are selected at random. Calculate the probabilities that the selected group will contains i) no girls ii) only one girl iii) only one particular girl iv) at least one girl v) more girls than boys. (Ans: 5/28, 15/28, 5/28, 23/28 and 2/7)

9. The probability that a person selected at random from a population will exhibit the classic symptom of certain disease is 0.2, and the probability that a person selected at random has the disease is 0.23. The probability that a person who has the symptom also has the disease is 0.18. A person selected at random from the population does not have the symptom: what is the probability that the person has the disease?

10. In a certain population women 4 percent have had the breast cancer, 20 percent are smokers and 3 percent are smokers and have had breast cancer. A woman is selected at random from the population, what is the probability that she has a breast cancer or smoker or both?

11. In a test of irregular heart beat it was found that probability of irregular heart beat is 20%. The probability of positive result given that regular heart beat is 0.3 and probability positive result given that the irregular heart beat is 0.7. Find the probability of irregular heart beat given that positive result.

12. Suppose the probability that a person has certain disease is 0.03. Medical diagnostic test are available to determine whether the person actually has disease. If the disease is actually present the probability that the medical diagnostic test will give positive result is 0.09. If the disease is not actually present the probability of positive test result is 0.02. Suppose the medical diagnostic test has given a positive result. What is the probability that the disease is actually present?

13. Suppose that a person visits his dentist, the probability that he will have his teeth cleaned is 0.44, the probability that he will have cavity filled is 0.24, the probability that he will have tooth extracted is 0.21, the probability that he will have a teeth cleaned and cavity filled is 0.08, the probability that he will have his teeth cleaned and tooth extracted is 0.11, the probability that he will have his cavity filled and tooth extracted is 0.07 and the probability that he will have his teeth cleaned, cavity filled and tooth extracted is 0.03. What is the probability that a person visiting his dentist will have at least one of the things done to him?

14. Suppose there were a test for cancer with the property of 90% of these with cancer reacted positively where as 5% of these without cancer react positively. Assume that 1% of patients in a hospital have a cancer. What is the probability that a patient selected at random who reacts positively to this test actually has a cancer?

15. The human resource department of a company has records which show the following analysis of 200 pharmacists.

Age	Bachelor's degree	Master's degree
Under 30	90	10
30 – 40	20	30
Over 40	40	10

If one of the pharmacist is selected at random from the company, find the probability that i) he has only a bachelor's degree ii) he is master degree given that he is over 40.

16. Following data represents the age and marital status of 140 people.

Age	Single	Married
Under 35	77	14
35 and more	28	21

If one person is selected at random find the probability that i) he is single and under age 35 ii) he is under 35 given that he is single.

17. The probability that a boy will be HIV positive in ELSA test is 0.75 and that a girl will be HIV positive is 0.72. What is the probability that i) both will be HIV positive ii) at least one of them will be HIV positive?

18. In a certain population of hospital patients, the probability is 0.35 that a randomly selected patient will have heart disease. The probability is 0.86 that the patients with heart disease are smokers. What is the probability that a patient randomly selected from population will be smoker and have a heart disease?

19. A medical representative carries two medicines Ciprofloxacin and Flexon. He estimates that when he makes a call, 35% of time he sells Ciprofloxacin and 50% of time, he sells Flexon. He knows that she sale of two medicines are independent. i) what is the probability that he will sell both medicines on the same call? ii) what is probability that he will sell either Ciprofloxacin or Flexon but not both?

Random Variables and probability distribution

Consider a random experiment of tossing a coin 2 times. Let X be a real value function of getting head.

Outcome w:	HH	HT	TH	TT
Value of X:	2	1	1	0

Here 2 mean that X associates with 2 heads, 1 means X associates with 1 head and 0 means X associates with 0 head.

Definition: A random variable (r.v) is a real valued function X(w) defined on sample space S (domain) and range $(-\infty,\infty)$ in random experiment. Random variables are denoted by X,Y,Z and value of random variables are denoted by x,y,z.

There are two types of random variables they are:

i) Discrete random variable: A random variable is said to be discrete if it takes at most countable values i.e. fixed integer value. No. of patients arrived in a hospital, no. of death, no. of accidents etc are examples of discrete random variable.

Probability mass function: Let X be a one dimensional discrete random variable which associates with countable infinite no. of values $x_0, x_1, x_2,$.........and the probability of each x_i is P(X=x_i) = P(x_i). If the probability P(x_i), i = 0,1,2...... must satisfy the following conditions

i) $P(x_i) \geq 0,$ for i = 0, 1, 2, ... and ii) $\sum_{i=0}^{\infty} P(x_i) = 1$

Then the function P(x_i) or simply P is called probability mass function of the discrete r.v. X and the set of pair (x_i P(x_i)) is called probability distribution of the r.v X.

ii) Continuous random variable: A random variable X is said to be continuous if it can take all possible value between certain limits. Data relating to age, height, weight, blood pressure etc are examples of continuous random variables.

Mathematical Expectation:

Mathematical expectation of a discrete random variable:

Let X be a discrete random variable which associates with value $x_1, x_2, \ldots\ldots\ldots, x_n$ with respective probabilities $P(X = x_i)$, i = 1,2,.........,n, then its mathematical expectation is defined as

$$E(X) = \sum_{i=1}^{n} x_i P(X = x_i) = \sum x_i p_i \quad , \qquad \sum p_i = 1$$

Theorems on expectation:

i) E(X+Y+Z+............+T) = E(X) + E(Y) + E(Z) +...............+E(T)

ii) If X, Y, Z,..........T are independent random variables then

E(X.Y.Z..........T) = E(X). E(Y). E(Z).........E(T)

iii) Expectation of constant is constant itself. i.e. E(a) = a

iv) If a and b are constants, then the variance of a r.v a function h(x) which is function of r.v X. i.e.

h(x) = aX + b

V(h(x)) = V(aX + b) = V(aX) + V(b) = $a^2 V(X)$ $\quad \because$ V(constant) = 0

Variance of a random Variable:

Let X be a random variable with mean E(X) = μ, then variance of a random variable is given by

$$V(X) = E\{E(X) - E(X)\}^2 = E(X^2) - \{E(X)\}^2$$

Covariance: The measure of the simultaneous variation between the random variables X and Y is denoted by Cov(X,Y) and is defined as:

Cov(X,Y) = E{{X-E(X)}{Y – E(Y)}}

= E{XY –XE(Y) – YE(X) +E(X).E(Y)} = E(XY) –E(X)E(Y) – E(Y)E(X) + E(X)E(Y)

$\because$ Cov(X,Y) = E(XY) – E(X)E(Y)

If X and Y are independent then Cov(X,Y) = 0

Expectation of continuous random variable:

Let X be a continuous random variable with p.d.f f(x) then mathematical expectation is defined as

E(X) = $\int_{-\infty}^{\infty} xf(x)dx$

Ex: A random variable X has the following probability function.

Value of X: x	-2	-1	0	1	2	3
P(x)	0.1	k	0.2	2k	0.3	k

Find the mean and variance of the distribution.

Solution:

$\sum P(x_i) = 1$, $\therefore$ 0.1+k+0.2+2k+0.3+k =1 $\Rightarrow$ k = 0.1

Mean = E(X) = $\sum_{i=1}^{n} x_i P(x_i)$ = (-2)(0.1)+(-1)(0.1)+0+1(0.2)+2(0.3)+3(0.1) = 0.8

$E(X^2) = \sum_{i=1}^{n} x_i^2 P(x_i)$ = 4x0.1+1x0.1+0+1x0.2+2x0.3+9x0.1 = 2.8

V(X) = $E(X^2) - (E(X))^2 = 2.8 - (0.8)^2 =$ 2.16

Exercise:

1. Calculate the mean and variance for the following probability distribution.

a) X: 10 20 30 40 50

P(X): 0.12 0.32 0.26 0.18 0.12

b) X: -1 0 1 2

P(X): 1/3 1/6 1/6 1/3

2. Suppose X is a random variable having the p.m.f p(x) is given by:

X:x	-3	-1	0	1	2	3	5	8
P(x)	0.1	0.2	0.15	0.2	0.1	0.15	0.05	0.05

Find the mean and variance.

3. A probability distribution is given by:

X:	0	1	2	3	4	5
P(x):	0.26	0.25	0.11	0.02	0.25	0.11

Find i) P(4) ii) P(0<X< 4) iii) P(X $=4 \cap X=5$) (independent cases) iv) find mean and variance.

4. The following table represents the probability distribution of the number visits to the doctor by an insured individual during a period of one year.

No. of visits :	0	1	2	3	4	5	6
Probability:	k	0.15	2k	3k	k	0.07	0.08

Find the value of k and mean and variance.

5. A random variable X has the following probability function:

X :	0	1	2	3
P(x):	0.1	0.2	0.3	0.4

Find the probability function of Y and mean of Y where Y = $x^2 + 1$

6. DISCRETE PROBABILITY DISTRIBUTION

Binomial Distribution

Binomial Experiment: A binomial experiment (also known as a Bernoulli trial) is a statistical experiment that has the following properties:

- The experiment consists of *n* repeated trials.
- Each trial can result in just two possible outcomes. We call one of these outcomes a success and the other, a failure.
- The probability of success, denoted by *P*, is the same on every trial.
- The trials are independent; i.e., the outcome on one trial does not affect the outcome on others.

Definition: A discrete random variable X is said to follow binomial distribution if its probability mass function is given by

$$P(X=x) = n_{C_x} p^x q^{n-x} = \frac{n!}{(n-x)!x!} p^x q^{n-x}, \qquad x = 0,1,2\ldots\ldots\ldots n$$

= 0 otherwise

Where n and p are also called parameter of the distribution. P = probability of successes, q = probability of failure and x = number of successes in trail.

Mean E(X) = np, Variance = npq

Eg: In a certain developing country 30% children are undernourished. From a random sample of 6 children in this area, find the probability that number of undernourished are i) exactly two ii) More than 2 iii)at least 2 iv) at most 2. Also find the mean and standard deviation of binomial distribution. If there are 50 groups of children, in how many groups more than 2 children are undernourished?

Solution:

Probability of undernourished (p) = 30% = 0.3, sample size (n) = 6, q = 1-p = 0.7. The distribution follows binomial so,

i) P(x=2) = $n_{C_x} p^x q^{n-x} = 6_{C_2} (0.3)^2 (0.7)^{6-2}$ = 0.3241

ii) P(x>2) = P(x=3) + P(x=4) + P(x=5) + P(x=6)

$$= 6_{C_3} (0.3)^3 (0.7)^{6-3} + 6_{C_4} (0.3)^4 (0.7)^{6-4} + 6_{C_5} (0.3)^5 (0.7)^{6-5} + 6_{C_6} (0.3)^6 (0.7)^{6-6}$$

= 0.1852 + 0.0595 + 0.010 + 0.0007 = 0.2554

iii) P($x \geq 2$) = P(x=2) + P(x=3) + P(x=4) + P(x=5) + P(x=6) = 0.3241 + 0.2554 = 0.57975

Alternately,

P(x≥ 2) = 1- P(x<2) = 1 - {P(x=0) + P(x=1)}

iv) P(x$\leq$2) = P(x=0) + P(x=1) + P(x=2)

$$= 6_{C_0}(0.3)^0(0.7)^{6-0} + 6_{C_1}(0.3)^1(0.7)^{6-1} + 6_{C_2}(0.3)^2(0.7)^{6-2} = 0.7442$$

Mean of the distribution (μ) = np = 6x0.3 = 1.8

Variance(σ^2) = npq = 6x0.3x0.7 = 1.26, $\sigma = \sqrt{npq}$ = 1.12

If there are 50 group of children, then more than 2 children are undernourished are

N.P(x>2) = 50x0.255 = 12.77 = 13

Approximately 13 group of children have more than 2 children undernourished.

Eg: A recent study by the traffic police revealed that 60% of the car drivers wearing seat belts. A sample of 10 drivers on new road is selected as pilot survey, find the probability that i) exactly 7 are wearing seat belt ii) 7 or fewer of the drivers are wearing seat belt?

Solution: The situation the meets the requirements of binomial distribution.

Sample size (n) = 10, probability of wearing seat belt (p) = 60% = 0.6, q = 0.4

In binomial distribution probability of getting x successes out of n trails is P(x) = $n_{C_x} p^x q^{n-x}$

i) P(x =7) = $10_{C_7}(0.6)^7(0.4)^{10-7}$ = 120 x 0.02799 = 0.215

ii) $P(x \le 7) = 1 - \{P(x>7)\} = 1 - \{P(x=8) + P(x=9)+P(x=10)\}$

$= 1 - \{0.120 + 0.040 + 0.006\} = 0.833$

Eg: A multiple test has 5 questions. There are 4 choices for each question. A student who has not studied for test decided to answer all the questions randomly. What is the probability that he will get i) five questions correct ii) at least 4 questions correct?

Solution: n = 5, probability of getting correct answer (p) = 1/4 = 0.25 , q = 0.75

i)$P(x=5) = 5_{C_5}(0.25)^5(0.75)^{5-5}$ = 1 x 0.00097 x1 = 0.00097

ii) $P(x \ge 4) = P(x=4) + P(x=5) = 5_{C_4}(0.25)^4(0.75)^{5-4} + 5_{C_5}(0.25)^5(0.75)^{5-5} =$ 0.01562

Exercise:

1. A production manager knows that 5% of components produced by a particular manufacturing process is defective. Six of these components, whose characteristics can be assumed to be independent of each are examined, what is the probability that i) none of these components has a defect ii) at least two of these components have a defect? (Ans: 0.735, 0.03277)

2) It is known from experience that 80% of the patients survive a particular heart surgery. In a year 18 patients were operated upon. What is the probability that i) 15 patients will survive ii) at least 15 patients will survive? (Ans: 0.23, 0.501)

3) In a four ICDS projects 20% of children under age 6 years were found to be malnourished. If 4 children are selected at random, what is the

probability that i) no children are malnourished ii) at least 2 children are malnourished.

4) The incidence of occupational disease in an industry is such that the workers have 20% chance of suffering from it, what is the probability that out of 6 workmen 4 or more will contact the disease?

5) Suppose that 30% of certain population is immune to some disease. If a random sample of size 10 is selected from this population, what is the probability that it will contain exactly 4 immune persons? (Ans:0.2001)

6) A National Center for Health Statistics report based on 1985 data states that 30% of American adults smoke. From the data 15 adults are selected at random, find the probability that the number of smokers in the sample would be i) less than five ii) between five and nine inclusive iii) more than five, but less than ten iv) six or more

7) The probability that a person suffering from migraine headache will obtain relief with a particular drug is 0.9.Three randomly selected sufferers from the migraine headache are given the drug. Find the probability that the number obtaining relief will be i) exactly one ii) more than one iii) at most two iv) two or three? (Ans: 0.27, 0.972, 0.271, 0.982)

8) Medical records show that one out of 10 persons in a certain town has thyroid deficiency. If 12 persons in this town are randomly selected and tested, what is the probability that at least one of them have a thyroid deficiency?

Poisson Distribution

Poisson distribution is considered as the limiting case of binomial distribution under the following conditions.

-The number of trails (n) is indefinitely large. i.e. n $\to \infty$

-The probability of successes (p) is indefinitely small. i.e p $\to 0$

-The product n and p is finite (say λ).

Definition: A random variable X is said to follow Poisson distribution if its probability mass function is given by

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \quad x = 0,1,2,3\ldots\ldots\ldots\infty$$

$$= 0, \text{ otherwise.}$$

Where λ = mean, x = number of successes, e = 2.71828 is the base of natural logarithm. Variance = λ

Eg: It is known from the past experience that in a certain plant there are 4 industrial accidents per year. Find the probability that in given year, there will be i) less than 4 accidents ii) at least 4 accidents iii) exactly 2 accidents iv) no accidents. If there are 200 workers in industry, how many of them will have exactly 2 industrial accidents?

Solution: Mean (λ) = 4, Lest x be the no. of accidents per year in the plant, its probability is given as

$P(x) = e^{-\lambda}\lambda^{x} / x!$

i) P(x< 4) = P(x=0) + P(x=1) + P(x=2) + P(x=3)

=

$= e^{-4}4^{0}/0! + e^{-4}4^{1}/1! + e^{-4}4^{2}/2! + e^{-4}4^{3}/3!$

$= e^{-4}(1/1 + 4/1 + 16/2 + 64/6)$

= 0.0183 x 23.67 = 0.4332

ii) $P(x \geq 4) = 1 - P(x < 4)$

$= 1 – 0.4332 = 0.5668$

iii) P(x = 2) = $e^{-4} 4^2 / 2!$ = 0.1464

iv) P(x = 0) = $e^{-4} 4^0 / 0!$ = 0.183

When N =200,

The number of workers with 2 industrial accidents = N. P(x = 2) = 200 x 0.1464 = 29.28

Required no. of workers is approximately = 29.

1) Assume that the chance of an individual coal miner being killed in mine accidents during a year is 1/1400. Calculate the probability that in a mine employing 350 miners there will be at least one fatal accident in a year.

2) A hospital switch board receives an average of 3 emergency calls per minute. What is the probability of receiving more than 2 calls?

3) Suppose the number of people seen for violent asthama attacks in the emergency ward of a hospital over a one day period is usually Poisson distributed with parameter lamda = 1.5. What is the probability of observing 5 or more cases in any specific day?

4) In a certain population on average 13 new cases of esophageal cancer are diagnosed each year. If the annual incidence of esophageal cancer follows a Poisson distribution, find the probability that in a given year the number of newly diagnosed cases of esophageal cancer will be at least three.

5) Past experience shows that an average number of 6 patients per day arrive at the health post at the case of diarrhea. What is the probability that i) 3 patient with diarrhea arriving any day ii) 3 or less cases of

diarrhea arriving in a given day iii) What is the expected value and standard deviation of the distribution?

6) Suppose that over a period of several years the average no. of deaths from certain noncontiguous disease has been 10. If the no. of deaths from this disease follows the Poisson distribution, what is the probability that during the current year seven people will die from the disease?

7) Assuming that one in 100 births is a case of twins appears and in a certain town on a day when 40 births occurs. What is the probability that two or more sets of twins will born in that day?

(Hints: p=1/100 = 0.01, λ = np = 40 x 0.01 then $P(x \geq 2)$ =?

8) In a certain desert the no. of persons who become seriously ill in each year from eating a certain poisonous plant is a random variable follows Poisson distribution, with λ=5.2. Find the probability of such 3 illness in a given year.

9) In the study of certain aquatic organism, a large number of samples were taken from a pond, and the number of organisms in each sample was counted. The average number of organisms per sample was found to be two. Assuming that the number of organisms follows a Poisson distribution, find the probability that the next sample will contain one or fewer organisms.

10) The mean number of serious accidents per year in a large factory (where the no. of employees remain constant) is five, find the probability that in the current year there will be i) no accidents ii) at most five accidents iii) at least 3 accidents.

CONTINUOUS PROBABILITY DISTRIBUTION:

The Normal Distribution

A continuous random variable X follows a **normal distribution** if it has the following probability density function (p.d.f.):

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \quad , -\infty < x < \infty$$

= 0 otherwise

Mean = μ, and variance = σ^2

Formula for the Standardized Normal Distribution

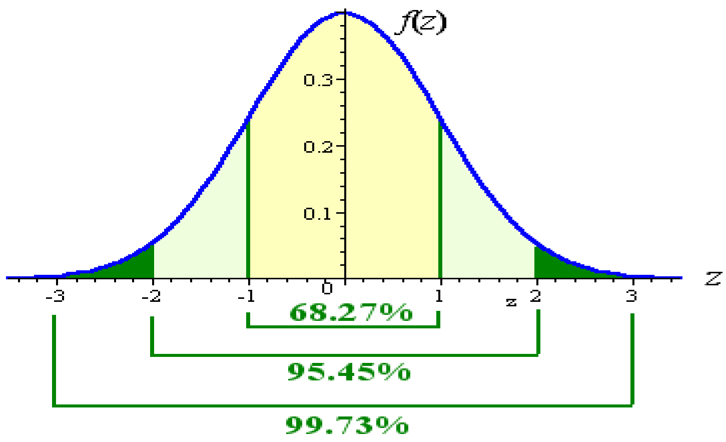
X~ N(μ , σ^2), then introduce a new variate Z, whose mean (μ)=0 and variance σ^2 =1,then

$$z = \frac{X-\mu}{\sigma} \qquad Z \sim N(0,1)$$

Here Z is called standard normal variate.

Percentages of the Area under the Standard Normal Curve

A graph of this standardized (mean 0 and variance 1) normal curve is shown.



In this graph, we have indicated the areas between the regions as follows:

$-1 \leq Z \leq 1 = 68.27\%$

$-2 \leq Z \leq 2 = 95.45\%$

$-3 \leq Z \leq 3 = 99.73\%$

This means that 68.27% of the scores lie within 1 standard deviation of the mean, comes from

$\int_{-1}^{1} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = 0.68269$ and so on for other. The total area from $-\infty < z < \infty$ is 1.

Characteristics of normal Curve:

*The curve is bell shaped and symmetrical about mean μ.

*The distribution is unimodel since the normal curve has only one peak.

*The value of the mean, median and mode are same (i.e. mean = median = mode).

* The two tails of the normal distribution extended indefinitely and never touch the horizontal axis. This property is called the asymptotic property of the curve.

The *z*-Table

The areas under the curve bounded by the ordinates $z = 0$ and any positive value of z are found in the ***z*-Table**. From this table the area under the standard normal curve between any two ordinates can be found by using the symmetry of the curve about $z = 0$.

Eg: Let X be normal variate with mean 15 and variance 16. Find out the probability of the following. i) $X \leq 12$) ii) $X \geq 17$ iii) $0 \leq X \leq 20$ iv) Find a when P(X>a) = 0.36

Solution: We have $\mu = 15$, $\sigma = \sqrt{16} = 4$,

i)when X =12, then

$Z = \frac{X - \mu}{\sigma}$ = (12 -15)/4 = -0.75

$P(Z \leq -0.75) = 0.5 - P(-0.75 \leq Z \leq 0)$

$= 0.5 - P(0 \leq Z \leq 0.75)$ $\because$ symmetricity of normal curve

= 0.5 – 0.2734 (from normal table)

= 0.2266

ii) When X=17, then

Z = (17-15)/4 = 0.5

$P(Z \geq 0.5) = 0.5 - P(0 \leq Z \leq 0.5)$

$= 0.5 - 0.1915$ (from normal curve)

$= 0.3085$

iii) $P(0 \leq X \leq 20) = ?$

When X=0, then

$Z = (0 - 15)/4 = -3.75$

When X = 20, then $Z = (20 - 15)/4 = 1.25$

$P(-3.750 \leq Z \leq 1.25) = P(-3.75 \leq Z \leq 0) + P(0 \leq Z \leq 1.25)$

$== P(0 \leq Z \leq 3.75) + P(0 \leq Z \leq 1.25)$ $\because$ symmetricity of normal curve

$= 0.499 + 0.394 = 0.893$

iv) $P(X>a) = 0.36$

when X =a, then

$Z = (a - 15)/4$

$P(Z > (a-15)/4) = 0.36$

Or, $0.5 - P(0 < Z < (a-15)/4) = 0.36$

Or, $P(0 < Z < (a-15)/4) = 0.1400$

$Z = 0.35$ (from normal table)

i.e. $(a-15)/4 = 0.35$

$\therefore$ a= 16.4

Eg: The weight of certain population of individuals is assumed to follow normal distribution with mean 140 pounds and standard deviation of 25 pounds i) what is the probability that a person picked at random from this group will weigh between 100 and 170 pounds ii) out of 10,000 persons how many would weight more than 200 pounds?

Solution: μ = 140 pounds , σ = 25 pounds

i) $100 \leq X \leq 170$

When X =100, then

$Z = (X - \mu)/\sigma = (100-140)/25 = -1.6$

When X=170

$Z = (170-140)/25 = 1.2$

$P(-1.6 \leq Z \leq 1.2) = P(-1.6 \leq Z \leq 0) + P(0 \leq Z \leq 1.2) = 0.4452+0.3849 = 0.8301$

ii) X = 200

$Z = (200 – 140)/25 = 2.4$

$P(Z \geq 2.4) = 0.5 – P(0 \leq Z \leq 2.4) = 0.5 – 0.4918$ From normal table

$= 0.0082$

No. of persons weight more than 200 pounds = 10,000 x 0.0082 = 82 persons.

EXERCISE :

1) In an experiment 15% of candidate got first class (60 or more) while 40% failed (less than 40) marks. Assuming the marks are normally distributed estimate the mean and s.d.

2) A diastolic blood pressure for a group of women aged 25b to 34 years are normally distributed with a mean of 82 mmHg and standard

deviation 3 mmHg. If a person is selected at random from this age group of women, what is the probability that her blood pressure i) lies between 77.5 to 85 mmHg ii) is less than 86.5?

3) A hospital records the weight of every new born child at the hospital. The distribution of weight is normally distributed and has the mean of 2.9 kg and standard deviation 0.45. i) Find the percentage of new born babies whose weight is between 1.8 kg and 4.0 kg. ii) If 1500 babies have been born at the hospital, how many of them have a weight less than 2.5 kg.

4) Suppose the ages at time of onset of certain disease are approximately normally distributed with mean of 11.5 years and standard deviation of 3 years. A child has just come down with the disease. Find the probability that the child is i) over 10 years of age ii) between 8 to 14 years of age.

5) The weights of new born babies at a particular hospital have been observed to be normally distributed with a mean of 7.4 pounds and standard deviation of 0.4 pounds. What is the probability that a baby born in this hospital will weight i) More than 8 pounds ii) less than 7 pounds?

6) The weight of the tablet of certain drug approximately follows normal distribution with mean 200 mg and standard deviation of 10 mg. If a tablet is selected at random find the probability that the weight of the tablet i) 185 and 200mg ii) less than 180 mg iii) more than 220mg/

7) Suppose it is known that the heights of a certain population of individuals are approximately normally distributed with a mean of 70 inches and a standard deviation of 3 inches. What is the probability that a person picked at random from the group will be between 65 and 74 inches?

8) If the capacities of the cranial cavities of a certain population are approximately normally distributed with a mean of 1400cc and standard deviation of 125, find the probability that a person randomly picked from this population will have a cranial cavity capacity i) Greater than 1450cc ii) Less than 1350cc iii) Between 1300 and 1500 cc. (Ans: 0.344, 0.344, 0.576

9) Suppose the average length of stay in a chronic disease hospital of a certain type of patient is 60 days with s.d. of 15. If it is reasonable to assume an approximately normal distribution of lengths of stay, find the probability that a randomly selected patient from this group will have a length of stay: i) Greater than 50 days ii) less than 30 days iii) Between 30 and 60 days.

10) If the total cholesterol values for a certain population are approximately normally distributed with a mean of 200mg/100ml and a standard deviation 20mg/100ml. Find the probability that an individual picked at random from this population will have a cholesterol value i) between 180 and 200mg/100ml ii) Greater than 225mg/100ml iii) less than 150mg/100ml (Ans: 0.341, 0.016, 0.771)

11) The weights of a certain population of young adult females are approximately normally distributed with a mean of 132 pounds and a standard deviation of 15. Find the probability that a woman selected at random from this population will weight i) more than 155 pounds ii) 100 pounds or less iii) Between 105 and 145 pounds. (Ans: 0.065, 0.016, 0.771)