

shows sig... the non-anesthetic group.

Wilcoxon Matched Pairs Test (Signed rank test: for dependent samples)

Frank Wilcoxon, USA was a chemist who wanted to develop a test similar to t-test but without requirement of Normal distribution. He developed the test and named as Wilcoxon Signed Rank test which is equivalent to paired t-test. It is an alternative method to test the paired data when the observations do not follow the criteria of normality.

It can be used to test paired data such as: Before and after studies, Studies of twins or other relatives, two different medicines used on the same groups at two different time periods, etc.

Each subject produces two scores, one for each condition.

Test is done to show whether there is a statistically significant difference between the two conditions.

Steps of calculation

- First find the differences(d) of the scores of the two matched samples.
- Assign the rank to the differences (ignoring the sign).
- Positive ranks are summed.
- Negative ranks are summed.
- T is the smaller sum of ranks.
- n is the number of matched pairs.

Conditions of test

- If $n > 15$, T is approximately normally distributed, and a Z test is used.
- If $n < 15$, a special "small sample" procedure is followed.
- The paired data are randomly selected.
- The underlying distributions are symmetrical

Hypothesis

Null hypothesis (H_0): The two populations are identical.

Alternative hypothesis (H_1): The two populations are not identical.

Example 1:

The data in the table below shows the duration of tolerance of pain by 11 subjects before and after the administration of a drug (0.04mg/20g). Does the data provide sufficient evidence in support that the drug increases the duration of endurance of pain?

Calculation of Wilcoxon's Signed Ranked Test (Small sample)

SN	Before Drug	After Drug	difference	Rank difference	Ranked with signs
1	154.5	21.2	-133.3	8	-8
2	12.7	20.1	7.4	11	+11
3	14.8	17.2	2.4	7	+7
4	16.7	22.7	6	9	+9
5	20.1	20	-0.1	1	-1
6	22	19.8	-2.2	6	-6
7	20.2	19.8	-0.4	3	-3
8	18.1	18.8	0.7	5	+5
9	17.6	17.9	0.3	2	+2
10	17.4	24.3	6.9	10	+10
11	19.1	18.6	-0.5	4	-4

Sum of negative ranks = -1 - 6 - 3 - 4 = -14

Sum of positive ranks = 8 + 11 + 7 + 9 + 5 + 2 + 10 = 52

The null hypothesis is tested using the smaller value of the sums of negative ranks (T). In this case, the sum of negative ranks (T) = 14

Tabulated value (critical value) T at 5% level of significance at 11 pairs = 11

Note: If the difference of the pair is zero, that pair should be subtracted from the total pairs.

Conclusion

Experimental value of T is 14, while tabled value of T is 11. It means that the null hypothesis can be ~~rejected~~ ^{accepted}. So, two populations are not identical.

Example 2:

Sleeping hours of the patients are studied to see whether the medicine worked or not. A particular drug and placebo are administered to 10 patients in separate days. The sleeping hours of these patients are recorded.

Patient	Hours of sleep	
	Drug	Placebo
1	6.1	5.2
2	7.0	7.9
3	8.2	3.9
4	7.6	4.7
5	6.5	5.3
6	8.4	5.4
7	6.9	4.2
8	6.7	6.1
9	7.4	3.8
10	5.8	6.3

Solution

Null Hypothesis: Hours of sleep are the same using placebo & the drug.

Alternative hypothesis: Hours of sleep are significantly different using placebo & the drug.

Patient	Hours of sleep		Difference	Rank Ignoring sign
	Drug	Placebo		
1	6.1	5.2	0.9	3.5*
2	7.0	7.9	-0.9	3.5*
3	8.2	3.9	4.3	10
4	7.6	4.7	2.9	7
5	6.5	5.3	1.2	5
6	8.4	5.4	3.0	8
7	6.9	4.2	2.7	6
8	6.7	6.1	0.6	2
9	7.4	3.8	3.6	9
10	5.8	6.3	-0.5	1

Sum of negative ranks = -1 - 3.5 = -4.5

Sum of positive ranks = 3.5 + 10 + 7 + 5 + 8 + 6 + 2 + 9 = 50.5

The null hypothesis is tested using the smaller value of the sums of negative ranks (T). In this case, the sum of negative ranks (T) = 4.5

Tabulated value (critical value) T at 5% level of significance at 11 pairs = 8

Here, calculated value of T = 4.5 is less than the tabulated value of T = 8 (at 5%). So, we reject the null hypothesis. Therefore, it is significant at 5% level indicating that the drug is more effective than placebo.

Interpretation

Because the test was significant, we accept the Null Hypothesis that the medians of the two conditions are significantly different. The result could be reported in APA format as follows. A Wilcoxon Matched Pairs T-test indicated significant difference between the placebo and experimental conditions, $T(N = 11) = 4.5, p > .05$.

Example 3:

Eight patients suffering from OCD were exposed to both a Placebo and an Experimental treatment (at different times). They were measured on severity of symptoms, on a 20-point scale where 20 = highest severity. Previous research indicates that severity of symptoms is not normally distributed in this population. Was there a significant difference in severity of symptoms between the two conditions?

Placebo	:	10	14	12	15	6	8	9	14
Experimental	:	8	10	13	15	4	11	6	12

Solution:

Step 1. Find the difference of score for each pair of scores. We can subtract in either direction, but we must use the same direction all the way through the data.

Step 2. Rank the D scores from lowest to highest in absolute value, leaving out any D scores of 0. When two or more D scores have the same absolute value, average their ranks and assign the average to each of the tied scores. For example, if three scores are tied for ranks 2,3 and 4, each of them receives a rank of 3 (average of 2,3 and 4).

Placebo	Experimental	D= difference	Absolute value of D	Sign
10	8	+2	2	Positive
14	10	+4	4	Positive
12	13	-1	1	Negative
15	15	0		
6	4	+2	2	Positive
8	11	-3	3	Negative
9	6	+3	3	Positive
14	12	+2	2	Positive

Step 3. Sum the positive ranks and the negative ranks.

$$\Sigma R \text{ positive} = 3 + 7 + 3 + 5.5 + 3 = 21.5.$$

$$\Sigma R \text{ negative} = 1 + 5.5 = 6.5 \quad T = 6.5$$

The null hypothesis is tested using the smaller value of the sums of negative ranks (T). In this case, the sum of negative ranks (T) = 6.5

Step 4. Tabulated value (critical value) T at 5% level of significance at 7 pairs = 2

Here, calculated value of T = 6.5 is greater than the tabulated value of T = 2 (at 5%), we reject the null hypothesis. Therefore, the test is not significant.

Interpretation

Because the test was not significant, we fail to reject the Null Hypothesis that the medians of the two conditions are equal. The result could be reported in APA format as follows. A Wilcoxon Matched Pairs T-test indicated no significant difference between the placebo and experimental conditions, $T(N = 7) = 6.5, p < .05$.